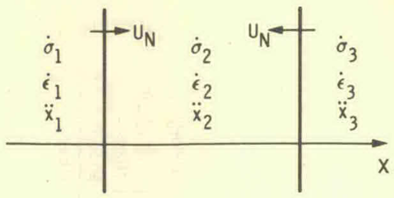

FIG. 2. Acceleration waves at the instant before collision.

In considering experimental capabilities it is important to observe that the theories are only concerned with situations in which significant unsteady behavior is expected. Hence, the interactions between characteristic material relaxation time, characteristic instrumentation time, and wave transit time between measuring stations must be explicitly considered. Most authors who have discussed implementation of the dual-wave theory[15] have implicitly assumed that instruments have the ideal capabilities of time resolution of a few nanoseconds in changes in velocity or stress, coupled with an *in situ* location at which the velocity or stress can be measured without perturbing the disturbance. Except in special cases these capabilities do not exist.

The ideal conditions can be obtained at present if a Sandia velocity interferometer[6] is employed to measure the velocity history of an imbedded mirror in an optically clear solid.[16] (This measurement does not satisfy the requirements of the dual-wave theory since only the velocity history is measured.) A close approximation to the ideal measurement is achieved when a velocity interferometer with a fused quartz window or a Sandia quartz gauge[17] is employed to measure the response of materials such as aluminum or beryllium which have good impedance matches to the gauge. When the velocity interferometer is employed with a sapphire window or a Sandia sapphire gauge[18] is employed to investigate iron, good time resolution and impedance matching are also obtained.

Techniques have been developed to imbed various gauges directly in solid samples. These gauges include a manganin gauge,[19,20] a dual velocity and stress gauge,[21] and an electromagnetic stress gauge.[22] However, experimental studies to date have demonstrated time resolutions of no better than about 50 nsec. Furthermore, the uniqueness of the gauge responses in various solid assemblies remains in question.

If materials with significant relaxation rates are to be investigated, time resolution of velocity or stress of 50 nsec leads to either insufficient accuracy in wave speed and amplitude measurements over short gauge spacings or insufficient accuracies in the amplitude changes between large gauge spacings. A recent paper[23] analyzes the unsteady behavior associated with elastic precursor decay, one of the most unsteady phenomena associated with shock-wave loading. The data analysis presented in that paper can be used as an explicit example of the interaction of unsteady behavior, measurements of rapidly changing amplitudes, and the dual-wave velocities. Although dual-wave speeds were not measured in the paper, reasonable assumptions concerning the constitutive relation were used to compute the speeds of velocity and stress waves. The difference is small (4% in the most unsteady case shown). Even though the difference in wave speed is small, a very large error is introduced if a gauge used to measure the local stress history has inadequate time-response capabilities. For example, data in the paper show that the stress amplitude changes 30% in a 50-nsec interval when the wave speeds differ by 4%. Thus, a gauge limited to a characteristic time of 50 nsec for accurate detection of changes in stress would incorporate errors in amplitude which are much greater than possible differences in wave speeds.

This example serves to emphasize that the most important consideration for the determination of unsteady behavior is the capability to accurately determine changes in amplitude with a time resolution of a few nanoseconds. The acceleration wave theory appears to provide an adequate description for most unsteady wave measurements accomplished with existing gauges.

APPENDIX: INTERACTIONS OF PLANE ACCELERATION WAVES OF UNIAXIAL STRAIN

The foregoing work has dealt with conditions that must be satisfied at isolated acceleration waves. In the following we examine the results of various kinds of acceleration wave interactions. These results on wave interactions are necessary if we are to use the theory in the interpretation of most kinds of experimental results, because it is usually necessary to account for the interaction of waves with each other inside samples, and with gauges or free surfaces at the boundary of samples.

Collision of two acceleration waves

At the instant before collision we have the situation shown in Fig. 2. The field variables in the initial state are constrained by their having to satisfy the jump conditions (5.2), which we write in the form

$$[\dot{\epsilon}]U_N + [\ddot{x}] = 0, \quad [\dot{\sigma}] + \rho_0 U_N [\ddot{x}] = 0.$$

Applying these conditions to each of the two waves we have

$$(\dot{\epsilon}_2 - \dot{\epsilon}_1)U_N + (\ddot{x}_2 - \ddot{x}_1) = 0, \quad (\dot{\sigma}_2 - \dot{\sigma}_1) + \rho_0 U_N(\ddot{x}_2 - \ddot{x}_1) = 0, \tag{A1}$$

$$(\dot{\epsilon}_3 - \dot{\epsilon}_2)U_N - (\ddot{x}_3 - \ddot{x}_2) = 0, \quad (\dot{\sigma}_3 - \dot{\sigma}_2) - \rho_0 U_N(\ddot{x}_3 - \ddot{x}_2) = 0.$$

Following the collision the situation is as shown in Fig. 3 and the jump conditions must again be satisfied at the waves, giving

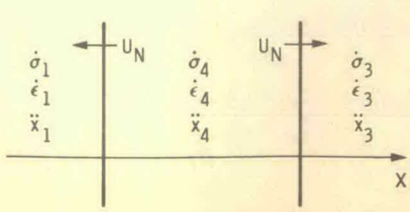

FIG. 3. Acceleration waves at the instant after collision.

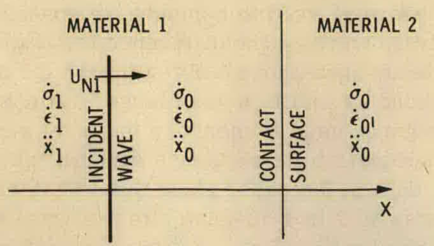

FIG. 4. Acceleration wave incident upon a contact surface.

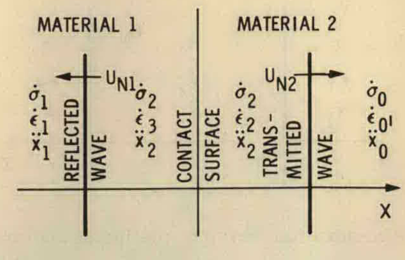

FIG. 5. Acceleration wave reflected from and transmitted through a contact surface.

$$(\dot\epsilon_4-\dot\epsilon_1)U_N-(\ddot x_4-\ddot x_1)=0,\quad (\dot\sigma_4-\dot\sigma_1)-\rho_0U_N(\ddot x_4-\ddot x_1)=0, \tag{A2}$$

$$(\dot\epsilon_3-\dot\epsilon_4)U_N-(\ddot x_3-\ddot x_4)=0,\quad (\dot\sigma_3-\dot\sigma_4)+\rho_0U_N(\ddot x_3-\ddot x_4)=0.$$

Solving these equations for $\dot\sigma_4$, $\dot\epsilon_4$, and $\ddot x_4$ we have

$$\begin{aligned}\dot\sigma_4&=\tfrac{1}{2}[(\dot\sigma_1+\dot\sigma_3)-\rho_0U_N(\ddot x_1-\ddot x_3)],\\ \dot\epsilon_4&=\tfrac{1}{2}[(\dot\epsilon_1+\dot\epsilon_3)-(1/U_N)(\ddot x_1-\ddot x_3)],\\ \ddot x_4&=\tfrac{1}{2}[(\ddot x_1+\ddot x_3)-(\dot\epsilon_1-\dot\epsilon_3)U_N].\end{aligned} \tag{A3}$$

Not all the quantities in the right members of these equations are independent; they must satisfy (A1). Using these equations we rewrite (A3) in the equivalent form

$$\dot\sigma_4=\dot\sigma_1-\dot\sigma_2+\dot\sigma_3,\quad \dot\epsilon_4=\dot\epsilon_1-\dot\epsilon_2+\dot\epsilon_3,\quad \ddot x_4=\ddot x_1-\ddot x_2+\ddot x_3. \tag{A4}$$

Interaction of an acceleration wave with a contact surface

At the instant before collision of an acceleration wave with a contact surface we have the situation shown in Fig. 4. In conformity with (5.1) and the definition of a contact surface, we have $\dot\sigma$ and $\ddot x$ continuous across the material interface. Any *a priori* relationship that may hold between $\dot\epsilon$ and $\dot\epsilon_0$ is dependent on the constitutive equation for the materials.

Application of the jump conditions to the incident wave gives

$$(\dot\epsilon_1-\dot\epsilon_0)U_{N1}+(\ddot x_1-\ddot x_0)=0,\quad (\dot\sigma_1-\dot\sigma_0)+\rho_{01}U_{N1}(\ddot x_1-\ddot x_0)=0. \tag{A5}$$

Following collision of the incident wave with the contact surface the situation is as shown in Fig. 5. Application of the jump conditions to the reflected and transmitted waves gives

$$(\dot\epsilon_3-\dot\epsilon_1)U_{N1}-(\ddot x_2-\ddot x_1)=0,\quad (\dot\sigma_2-\dot\sigma_1)-\rho_{01}U_{N1}(\ddot x_2-\ddot x_1)=0, \tag{A6}$$

$$(\dot\epsilon_0-\dot\epsilon_1)U_{N2}-(\ddot x_0-\ddot x_2)=0,\quad (\dot\sigma_0-\dot\sigma_2)+\rho_{02}U_{N2}(\ddot x_0-\ddot x_2)=0.$$

From these equations we find that

$$\dot\sigma_2=\frac{\rho_{01}U_{N1}\dot\sigma_0+\rho_{01}U_{N2}\dot\sigma_1-\rho_{01}U_{N1}\rho_{02}U_{N2}(\ddot x_1-\ddot x_0)}{\rho_{01}U_{N1}+\rho_{02}U_{N2}}, \tag{A7}$$

$$\ddot x_2=\frac{\rho_{01}U_{N1}\ddot x_1+\rho_{02}U_{N2}\ddot x_0+\dot\sigma_0-\dot\sigma_1}{\rho_{01}U_{N1}+\rho_{02}U_{N2}},$$

or

$$\sigma_2=\frac{2\rho_{02}U_{N2}\dot\sigma_1+(\rho_{01}U_{N1}-\rho_{02}U_{N2})\dot\sigma_0}{\rho_{01}U_{N1}+\rho_{02}U_{N2}},$$

$$\ddot x_2=\frac{2\rho_{01}U_{N1}\ddot x_1-(\rho_{01}U_{N1}-\rho_{02}U_{N2})\ddot x_0}{\rho_{01}U_{N1}+\rho_{02}U_{N2}},$$

and (A8)

$$\dot\epsilon_3=\frac{2\rho_{02}U_{N2}\dot\epsilon_1+(\rho_{01}U_{N1}-\rho_{02}U_{N2})\dot\epsilon_0}{\rho_{01}U_{N1}+\rho_{02}U_{N2}},$$

$$\dot\epsilon_2=\dot\epsilon_{01}+\frac{2\rho_{01}U_{N1}(U_{N1}/U_{N2})(\dot\epsilon_1-\dot\epsilon_0)}{\rho_{01}U_{N1}+\rho_{02}U_{N2}}.$$

For elastic materials, $U_1^2\dot\epsilon_0=U_2^2\dot\epsilon_{01}$, $U_1^2\dot\epsilon_3=U_2^2\dot\epsilon_2$.

Reflection at a free boundary

The free-surface reflection problem is a special case of the foregoing contact surface interaction problem in which the second material is replaced by a void. In this case

$$\dot\sigma_0=0,\quad \rho_{02}U_{N2}=0, \tag{A9}$$

and (A8) becomes

$$\dot\sigma_2=0,\quad \ddot x_2=2\ddot x_1-\ddot x_0,\quad \dot\epsilon_3=\dot\epsilon_0, \tag{A10}$$

Graphical solution of acceleration wave interaction problems

As we have seen, a plane acceleration wave of uniaxial strain in which the material passes from state 1 to state 2 must satisfy the jump condition (5.2):

$$(\dot\sigma_2-\dot\sigma_1)+\rho_0U_N(\ddot x_2-\ddot x_1)=0.$$

For a fixed state 1, this means that all accessible states 2 lie on a straight line in the $(\dot\sigma,\ddot x)$ plane. This line passes through $(\dot\sigma_1,\ddot x_1)$ and has slope $-\rho_0U_N$. The $(\dot\sigma,\ddot x)$ plane is particularly useful for considering wave interactions because the values of these quantities are continuous across contact surfaces.

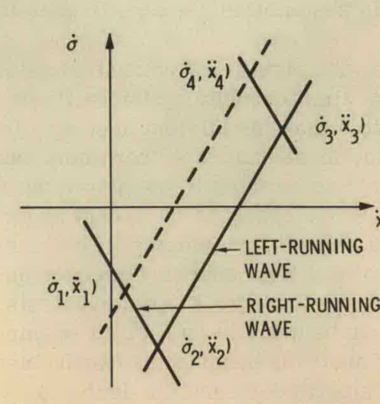

FIG. 6. Stress rate—acceleration diagram of acceleration wave collision with a contact surface.